Signal reconstruction and regulators

This manual page describes how the signal is reconstructed and the network constraints (often called regulators) are applied. Network vectors are indicated with the bold font. For example, **w[i] is** the K-component vector representing 0-phase data from K detectors at a time-frequency index i, which is always omitted below. K is the number of detectors in the network. The entire cluster 0-phase and 90-phase data is presented by a collection of vectors {**w**,**W**}={**w**[i1],..**w**[iN],**W**[i1],..**W**[iN]}, where N is the number of TF pixels. Also the following definition is used:

(0a) (**a**·**b**)=(**a**[i]·**b**[i]) – scalar product of **a[i]** and **b[i]**

- the network response to a GW signal is parametrized as

(1a) **s** = **Fp***h + e***Fx***H
(1b) **S** = -**Fp***H + e***Fx***h

where the noise scaled antenna pattern vectors {**Fp**,**Fx}** are defined by the selection of the coordinate frame in the wave plane. Here {**s**,**S**} are the 0^0 and -90^0 detector responses, (h,H) are the GW quadrature amplitudes (0,90)-degrees phase, e is the ellipticity of the wave. For this convention the sign of the quadruple product ([**s** x **S**] · [**Fp** x **Fx**]) is defined by e.

(2a) (**s**·**Fp**)(**S**·**Fx**)-(**S**·**Fp**)(**s**·**Fx**)   =
$e*(h^2+H^2)[(|\mathbf{Fp}||\mathbf{Fx}|)^2-(\mathbf{Fp}\cdot\mathbf{Fx})^2]$

- the cWB antenna patterns {**Gp**·**Gx**}

- the DPF antenna patterns (f+,fx) are defined as
(3a) **f+** = **Gp***c[d] + **Gx***s[d]; **Gp** = **Fp***c[p] - **Fx***s[p]
(3b) **fx** = **Gx***c[d] - **Gp***s[d]; **Gx** = **Fx***c[p] + **Fp***s[p]
where d is the DPF angle and p is the polarization angle, which defines the conversion between
{**Gp**,**Gx**}<->{**f+**,**fx**} and {**Gp**,**Gx**}<->{**Fp**,**Fx**}
antenna patterns respectively. The conversion between {**f+**,**fx**} and {**Fp**,**Fx**} is defined as

(4a) f+ = Fp*c[-p+d] + Fx*s[-p+d] = Fp*c + Fx*s
(4b) fx = Fx*c[-p+d] - Fp*s[-p+d] = Fx*c - Fp*s
(4c) Fp = f+*c[-p+d] - fx*s[-p+d] = f+*c - fx*s
(4d) Fx = fx*c[-p+d] + f+*s[-p+d] = fx*c + f+*s
In the second set of equations and below we drop the sin/cos argument -p+d

- The GW responses in the (f+,fx) frame are
  (5a) s = f+*(h*c+e*H*s) + fx*(-h*s+e*H*c)
  (5b) S = f+*(-H*c+e*h*s) + fx*(H*s+e*h*c)
  (5c) (s·f+)^2+(S·f+)^2 = |f+|^4 (h^2+H^2) (c^2+e^{2}s^2)
  (5d) (s·fx)^2+(S·fx)^2 = |fx|^4 (h^2+H^2) (s^2+e^{2}c^2)

- The standard likelihood analysis reconstructs signal as projections of the data vectors (w,W) on the network plane defined by the vectors f+ and fx.
  (6a) wp = (w·f+)/|f+|^2 ~ h*c+e*H*s
  (6b) wx = (w·fx)/|fx|^2 ~ -h*s+e*H*c
  (6c) Wp = (W·f+)/|f+|^2 ~ -H*c+e*h*s
  (6d) Wx = (W·fx)/|fx|^2 ~ H*s+e*h*c

- Therefore the GW responses are reconstructed as
  (7a) s = f+*(w·f+)/|f+|^2 + fx*(w·fx)/|fx|^2
  (7b) S = f+*(W·f+)/|f+|^2 + fx*(W·fx)/|fx|^2
  This solution is trivial for 2-detector networks:
  s=w and S=W

- To address the problem of trivial solutions, the signal constraints (model assumptions) and/or regulators SHULD BE applied. Below only the regulators are addressed – e.g. we consider un-modeled search with no assumptions on the source model. Lets define:

  (8a) E+ = (w·f+)^2+(W·f+)^2
  (8b) Ex = (w·fx)^2+(W·fx)^2
  (8c) E = (w·w)+(W·W) – data energy normalized by noise
  (8d) Eh = (h^2+H^2) – GW energy normalized by noise

- f+ regulator **delta**: the purpose of this regulator is to diminish signal reconstruction at sky locations with low network sensitivity that unlikely yield a detectable signal.

(9a) $[E+/E]^{1/2} < |\mathbf{f+}|^2 \, [Eh/E]^{1/2}$

(9b) $f^2 = \text{delta } [E+/E]^{1/2}$ if $\text{delta}*[E+/E] > |\mathbf{f+}|^2$

(9c) $f^2 = |\mathbf{f+}|^2$ if $\text{delta}*[E+/E] < |\mathbf{f+}|^2$

- fx regulator **R**: the purpose of this regulator is to diminish the reconstruction of hx component when |fx|<<|f+|

  (10a) $f^2[Ex/E+]^{1/2} = |\mathbf{fx}|^2 \, [(s^2+e^2c^2)/(c^2+e^2s^2)]^{1/2}$

  (10b) $F^2 = R \, f^2[Ex/H+]^{1/2}$ if $R \, f^2[Ex/E+]^{1/2} > |\mathbf{fx}|^2$

  (10c) $F^2 = |\mathbf{fx}|^2$ if $R \, f^2[Ex/E+]^{1/2} < |\mathbf{fx}|^2$

- Adaptive Regulator: R is defined by the network and pixel energy.

  (11a) count inverse fraction of the sky γ where $|\mathbf{fx}| > g^2$

  (11b) typical value of g^2 is 0.15-0.25

  (11c) $R = (\gamma^2-1)*Eo/E$, Eo is threshold for selection of the excess power pixels. When $g=0$, $\gamma=1$, $R=0$, $F^2=|\mathbf{fx}|^2$ – no regulator is imposed. When $g=1$ (max network sensitivity), $\gamma=R=F=\infty$ - this is the hard regulator. For $g=0.5$, $\gamma^2\sim2$ for advanced LHV network, $\gamma^2\sim300$ for O2 LH network and $\gamma^2\sim50$ for expected O2 LHV network.

- Reconstructed detector responses are

  (11a) $\mathbf{s} = \mathbf{f+}*(\mathbf{w}\cdot\mathbf{f+})/\mathbf{f}^2 + \mathbf{fx}*(\mathbf{w}\cdot\mathbf{fx})/F^2$

  (11b) $\mathbf{S} = \mathbf{f+}*(\mathbf{W}\cdot\mathbf{f+})/\mathbf{f}^2 + \mathbf{fx}*(\mathbf{W}\cdot\mathbf{fx})/F^2$

When F is large, only +-component of the signal is reconstructed.

Given reconstructed responses the signal packets are constructed as described in the wavelet packet manual.